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INVITED

ENERGY LOSSES IN CONDUCTORS CARRYING VERY HIGH CURRENTS

by

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Summary

Conductors carrying very high currents show losses of an electromagnetic and a shock compression nature. Electromagnetic losses (joule heating in the skin layer, magnetic flux diffusion) scale as H^3 or $(I/a)^3$, where H is the self magnetic field of the current and I/a is the current ÷ conductor periphery; shock losses scale as H^4 or H^3 $\{(I/a)^4$ or $(I/a)^3\}$ depending on the magnitude of I. In experiments where electrical energy must be converged from a large pulsed power supply to a small load, these losses can account for half the orignal energy and limit the magnitude of the energy per unit volume in the load. These considerations may be important for the study of material properties at high energy density.

The properties of materials in a state of high energy density if of interest to a broad range of scientific disciplines ranging from astrophysics to material science. Recent advances in pulsed power technology have made possible systems with energy storage in the megajoule range and have made possible the production of material energy densities ~ 1 MJ/cm^3.

A traditional approach to achieving high energy density is to obtain such a high energy pulsed power supply and design a "load" of arbitrarily small volume in order to attain a desired energy density. Where the energy store is in the form of charge or inductive storage, the load is connected to the energy storage device by a suitable transmission line; and the size of the line and the load are often constrained by mechanical and practical electrical considerations.

Since megajoule class energy storage systems are very large -- many meters in dimension -- and the load is often measured in centimeters, it is necessary to devise transmission lines which converge the flow of current from the source so that the electrical energy can be fed into the load. Near and at the load, the power convergence is maximum. It is the purpose of this paper to show that under such conditions, electromagnetic and shock compression losses in the conductors can account for a significant fraction of the total energy available. The losses scale as $(I/a)^3$ and $(I/a)^4$, where I/a is the current per unit length of current front (e.g., for a circular conductor, I/a ≡ current ÷ circumference). Since I/a is a function of the load geometry, there may exist an optimum load size for a maximum energy density in the load. Thus, the <u>highest</u> energy density is not produced by the smallest load volume. The presence of a current-carrying plasma sheet on the cathode side of the transmission line may produce a modest reduction in the magnitude of the loss, but should not affect the scaling with (I/a).

The losses we consider here derive from the penetration of the self magnetic field into the conductors as well as the associated ohmic heating of the current-carrying layer in the conductors. These we identify as the electromagnetic energy losses. We also consider losses due to the shock compression of the conductor surface layers (the shock is driven by the external magnetic pressure). Unless otherwise stated, the units are practical CGS. Unless otherwise stated, references to Knoepfel are to "Pulsed High Magnetic Fields," by H. Knoepfel (North Holland Publishing Company, 1970).

A. Electro Magnetic Energy Losses

From Poynting's theorem, the <u>total</u> electromagnetic energy per unit area flowing into the conductor (planar geometry) is

$$W_T = \frac{10^8}{4\pi}\int_0^t E H\,dt = -\frac{10^9}{(4\pi)^2}\frac{1}{\sigma}\int_0^t \frac{\partial H}{\partial x}\Big|_{x=0} H(o,t)\,dt$$

To obtain $\frac{\partial H}{\partial X}$ (o,t), one needs to solve the magnetic diffusion equation,

$$\nabla^2 H - \frac{1}{K}\frac{\partial H}{\partial t} = 0 \quad ; \quad K_0 = \frac{10^9}{4\pi\sigma\mu}$$

The solution depends on the time dependance of the magnetic field at the boundary surface. Generally, the pulse shape in inductance-dominated systems approximates a half-sinusoid; we approximate this with a time dependence given by

$$H(o,t) = H_0\left(\frac{t}{t_m}\right)^{1/2}$$

The integration over time implied in the energy equations would be over the interval $o < t < t_m$. For the proposed form of H (o,t), Knoepfel gives the solution as

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$$\frac{\partial H(x,t)}{\partial x}\Bigg|_{x=0} = -\frac{H_o\,\Gamma(3/2)}{\Gamma(1)}\left(\frac{t}{t_m}\right)^{1/2}\frac{1}{(K_o t)^{1/2}}$$

This analysis leads to the conclusions that

$$W_T = .047\,H_o^2(o,t)\,\sqrt{K_o t}$$

However, this treatment presumes that the conductivity σ is temperature independent. In fact, it varies more than an order of magnitude, e.g., for copper, the conductivity is ~ 10^6 (ohm cm)$^{-1}$ at room temperature and ~ 4.0×10^4 (ohm cm)$^{-1}$ at ~ 1500^oK.

Knoepfel (p. 88) obtains an approximate solution to the "nonlinear" (i.e., temperature dependent conductivity) problem for the case where the magnetic field is strong. He writes the applied field as

$$H(o,t) = h_c\sqrt{\frac{t}{T_o}}$$

and obtains solution valid if H >> h_c. Later, he clarifies the interpretation: h_c is that value of magnetic field where the heating due to the energy loss has caused the conductivity to drop by a factor of two. Thus, if the applied peak magnetic fields are generally below h_c, the conductivity variation with temperature is probably not important.

Since our interest here is scaling, we adopt the non-linear treatment of Knoepfel. The revised expression for the total electromagnetic energy loss is

$$W_T = .328\,\frac{H_m^3}{h_c}\sqrt{t_m} \qquad (erg/cm^2)$$

The calculations leading to this result are given in Appendix A.

There is loss mechanism in addition to joule heating and magnetic field penetration: current flowing in the conductor generates a magnetic field outside it; the magnetic pressure pushes on the conductor surface and generates a (weak) shock which propagates into the conductor. The material between the surface+the shock front is compressed, heated, and accelerated by the shock. The energy required to do this must come from the energy source and thus comprises a loss.

The treatment of this problem follows Knoepfel, p. 257, and the calculations leading to the energy loss per unit area are given in Appendix B. The result is, for a constant applied field,

$$W_{SH} = \frac{H^2}{16\pi}\,\frac{C}{S}\left[\sqrt{1+\frac{H^2 S}{2\pi\rho_o C^2}}-1\right]t_m \qquad (erg/cm^2)$$

ρ_o is the normal density of the conductor, and S and C are material properties. For copper,

S = 1.5

C = 4×10^5 cm/sec (~ speed of sound)

ρ_o = 8.9 g/cm^3

For very large currents, the magnetic field may be large enough to cause the second term in the square root to be significant. For the subsequent analyses, however, the second term has been dropped, and in this weak field approximation the shock loss per unit area is

$$W_{SH} = \frac{H^4 t_m}{64\pi^2\rho_o C} \qquad (erg/cm^2)$$

These results have been applied to the example of a solid current carrying copper rod. The magnetic field H = $\frac{2I}{10r}$ = 1.256 (I/2πr) = 1.256 (I/a). In Fig. 1, the loss in joules/cm^2 is plotted vs time for a variety of values of I/a. For times $\geq 10^{-7}$ sec, losses can be quite large in a practical system for I/a >.1 MA/cm.

We now apply these results to the design of a single turn coil of radius R and length Z inside which we wish to maximize the energy density. We assume the available energy is 1 megajoule and the inductance L_o of the feed line is 10 nh. However, for purposes of simplicity, we assume also that the width of the feed line is so great that losses in the line are negligible. R is assumed to be 1 cm, and we will determine the value of Z which maximizes the the internal energy density of the coil.

If Z >> R, the coil inductance is

$$L_c = \frac{4\pi^2 R^2}{100\,Z}10^{-7} = 3.95\times10^{-8}\frac{R^2}{Z} \quad (henries)$$

The losses are

$$W_T = .328\,\frac{H^3}{h_c}\,t_m^{1/2} \equiv \alpha H^3$$

$$W_{SH} = \frac{H^4 t_m}{64\pi^2\rho_o C} \equiv \beta H^4$$

We further assume H is constant and $t_m = 10^{-6}$ sec. The energy density inside the coil can be written as

$$\frac{E}{V} = \frac{E_o-(\alpha H^3+\beta H^4)\cdot 2\pi R Z}{\pi R^2 Z} \qquad (erg/cm^3)$$

If Z >> R, H = .4πI/Z,
and I can be obtained from

$$E_o = \tfrac{1}{2}(L_o+L_c)I^2 = 10^6\,joules$$

The expression for E/V takes the form

$$\frac{E}{V} = \frac{E_o}{\pi R^2 Z} - \frac{A_o}{(L_o Z+A_1)^{3/2}}\,\frac{1}{Z^{3/2}} - \frac{B_o}{(L_o Z^2+A_1)^2}\,\frac{1}{Z^2}$$

Table I shows how E/V varies with Z.

TABLE I

Z(cm)	E/V (joules/cm^3)
1	-1.14×10^7
2	-4.16×10^5
3	4.0×10^4
4	6.7×10^4
5	6.03×10^4
6	5.3×10^4

Thus, in this simplistic calculation, a coil length of Z ~ 4.5 cm gives the highest internal energy density of ~ 7×10^4 joules/cm^3, corresponding to H ~ 2.9

Megagauss. The loss is 10-15%, and increases rapidly if R or L_o is decreased or if E_o is increased.

Conclusions

In this paper, we have shown that conductors carrying high currents show two kinds of losses: electromagnetic losses, which scale as H^3, and shock compression losses, which scale as H^4 or H^3, where H is the self-field at the conductor surface. Since the loss per unit of conductor area depends on load geometry, the load volume cannot be made arbitrarily small to achieve the highest load energy density.

In a vacuum transmission line in which the transverse electric field is $\gtrsim 10^5$ v/cm, it is well known that a current carrying sheath forms at the cathode and about half the current flows in this sheath. If the losses in the sheath are negligible, the electromagnetic conductor losses in the cathode conductor inferred from the discussion of this paper are probably somewhat high. The losses at the anode conductor are unchanged. Since the current sheath is likely to be in pressure equilibirum with the self magnetic field and with the conductor surface, the losses due to shock compression are unaffected by the presence of the sheath.

Appendix A: Electromagnetic Energy Losses

As stated in the test, Poynting's theorem gives

$$W_T = -\frac{1}{\sigma_o}\frac{10^9}{(4\pi)^2}\int_0^t \frac{\partial H(x,t)}{\partial x}\Big|_{x=0} H(o,t)\,dt$$

To obtain the value of $\frac{\partial H}{\partial X}$ x=o, the magnetic diffusion equation must be solved; Knoepfel (p. 53; 76-77) obtains for <u>constant</u> conductivity

$$\frac{\partial H(x,t)}{\partial x}\Big|_{x=0} = -\frac{H_o\,\Gamma(3/2)}{\Gamma(1)}\left(\frac{t}{T_o}\right)^{1/2}\sqrt{\frac{1}{K_o t}}$$

where $H(o,t) = H_o\sqrt{\frac{t}{T_o}}$.

The total energy loss per unit area W_T is composed of two parts: joule heating and flux penetration into the surface. The latter is a loss because the flux cannot be recovered rapidly from the skin depth of the conductor. Denoting these two losses by W_J and W_M, respectively,

$$W_T = W_J + W_M$$

$$W_T = \frac{1}{8\pi}\int_0^\infty H^2(x,t)\,dx$$

$$W_J = \frac{1}{\sigma_o}\int_0^\infty dx\int_0^t\left[\frac{\partial H(x,t')}{\partial X}\right]^2 dt'$$

Knoepfel uses his solutions to the diffusion equation to obtain expressions for W_M and W_J:

$$W_M = \frac{H^2(o,t)}{8\pi}\,S_\phi\cdot 8\,\Gamma(3/2)\,\Gamma(2)\,I_1$$

$$W_J = \frac{H^2(o,t)}{8\pi}\,S_\phi\cdot\frac{8}{3}\,\Gamma(3/2)\,\Gamma(2)\,I_o$$

in this notation,

$$W_T = \frac{H^2(o,t)}{8\pi}\,S_\phi\cdot\frac{4}{3}$$

Here, I is an integral function defined by Knoepfel and and Γ (x) is the gamma function. S_ϕ is the flux penetration skin depth,

$$S_\phi = \frac{\Gamma(3/2)}{\Gamma(2)}\sqrt{K_o t}$$

(for these discussions, Knoepfel's "n" is 1.)
Knoepfel shows that the ratios W_M/W_T and W_J/W_T are

$$\frac{W_M}{W_T} = .414$$

$$\frac{W_J}{W_T} = .586$$

Thus, the ohmic and magnetic losses are nearly equal. W_T is given by

$$W_T = \frac{H^2(o,t)}{8\pi}\,\frac{4}{3}\,\frac{\Gamma(3/2)}{\Gamma(2)}\sqrt{K_o t} = .047\,H^2(o,t)$$

These results apply to the circumstance where σ, the material conductivity, is constant. However, σ is actually a function of temperature. For high currents and large magnetic fields, the surface layer can melt and even ablate, and the conductivity change can easily exceed a factor of two. Knoepfel (pp. 88-95) treats the "non-linear" problem (i.e., σ is temperature-dependent) for the case where the surface heating is a dominant effect. He starts by assuming

$$H(o,t) = h_c\sqrt{\frac{t}{T_o}}$$

where h_c is a characteristic field for the material. He shows that h_c is that magnetic field for which the implied surface heating causes the conductivity to drop by just a factor of two. For purposes of scaling to large magnetic fields or currents, the non-linear results are most appropriate.

Knoepfel's analysis shows how to modify S_ϕ for the non-linear case. He estimates in two ways the ratio of the non-linear S_ϕ to the quantity

$$\frac{H(o,t)\sqrt{K_o t}}{h_c}$$

and obtains

$$\frac{S_\phi}{\frac{H(o,t)\sqrt{K_o t}}{h_c}} = \frac{1}{3}\sqrt{2}\ \text{or}\ \frac{1}{2}\sqrt{2} = .471\ \text{or}\ .707$$

We will use the average of the numerical factors, 0.59. Thus,

$$W_T = \frac{H^2(0,t)}{8\pi}\,\frac{4}{3}\,S_\phi = \frac{H^2(0,t)}{8\pi}\,\frac{4}{3}\left\{\frac{.59\,H(0,t)\sqrt{K_0 t}}{h_c}\right\}$$

$$= \frac{.328\,H^3\sqrt{t}}{h_c} \quad \text{for copper}$$

This value will be used for further calculations. The division of this energy loss between joule dissipation and magnetic losses is the same as for the σ = constant case.

Appendix B: Shock and Compression Losses

The treatment of this problem follows Knoepfel, p. 257, and starts with statements of conservation of mass, momentum, and energy:

$$\rho_1\,(V_S - u) = \rho_o\,V_S$$

$$P_1 + \rho_1\,(V_S - u)^2 = P_o + \rho_o\,V_S^2$$

$$E_1 + \frac{P_1}{\rho_1} + \frac{1}{2}u^2 = E_o + \frac{P_o}{\rho_o}$$

(see Figure 2)

Here, ρ = density

V_S = shock velocity

u = material velocity

P = pressure

E = internal energy per unit mass

Subscripts 0,1 refer to unshocked and shocked material, respectively.

We assume that the material is in pressure balance with the magnetic field, i.e., (Fig. 2)

$$P_1 = \frac{H^2}{8\pi}$$

From the momentum conservation equation above,

$$P_1 = P_o + \rho_o V_s^2 - \rho_1 (V_s - u)^2 = \frac{H^2}{8\pi}$$

Since $P_o \ll P_1$, and from the mass conservation equation,

$$P_1 = \rho_o V_s^2 - \rho_o V_s (V_s - u) = \rho_o u V_s = \frac{H^2}{8\pi}$$

Empirically, it is known that a linear relationship between V_S and u agrees reasonably well with reality;

$V_S = C + Su$, C and S are material constants

So,

$$\rho_o u (C + Su) = \frac{H^2}{8\pi}$$

for which the solution is

$$u = \frac{C}{2S}\left[\sqrt{1 + \frac{H^2 S}{2\pi\rho_o C^2}} - 1\right]$$

The quantity $H^2 S/(2\pi\rho_o C^2)$ takes on the value 1 if $H \sim 2.5$ Megagauss. When $H \ll 2.5$ Megagauss, the weak field approximation can be used:

$$u = \frac{H^2}{8\pi\rho_o C}\,, \qquad \frac{H^2 S}{2\pi\rho_o C^2} \ll 1$$

when H > 2.5 Megagauss, the strong field approximation is useful:

$$u = \frac{H}{\sqrt{8\pi\rho_o S}}\,, \qquad \frac{H^2 S}{2\pi\rho_o C^2} \gg 1$$

The work done by the piston (the magnetic field) is, for a constant field,

$$W_{SH} = P_1 u t = \rho_o u^2 V_s t = \rho_o u^2 (C + Su) t$$

$$= \frac{H^2}{16\pi}\,\frac{C}{S}\left[\sqrt{1 + \frac{H^2 S}{2\pi\rho_o C^2}} - 1\right] t$$

For the strong field case, $\frac{H^2 S}{2\pi\rho_o C^2} \gg 1$, and

$$W_{SH} = \sqrt{\frac{S}{2\pi\rho_o}}\;\frac{H^3 t}{16\pi S}$$

In the weak field approximation, $\frac{H^2 S}{2\pi\rho_o C^2} \ll 1$, and

$$W_{SH} = \frac{H^4}{64\pi^2\rho_o C}\left[1 - \frac{S}{C^2}\,\frac{H^2}{8\pi\rho_o}\right]$$

Hence, for weak fields the loss scales as H^4 (or $I/a)^4$ but for larger fields, the scaling goes as H^3.

ENERGY LOSSES IN CIRCULAR CONDUCTORS

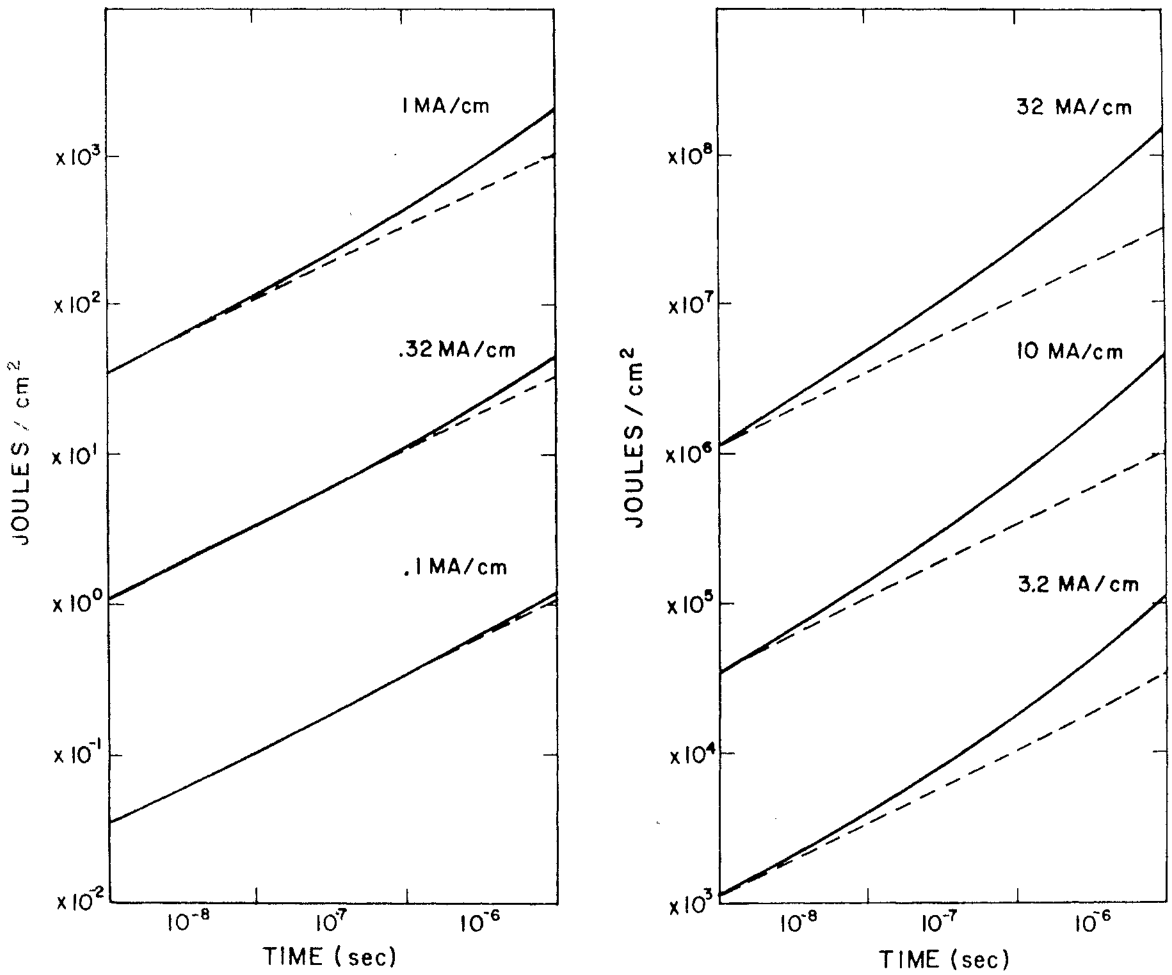

Figure 1. Losses in joules/cm^2 as a function of time for various values of $I/a = I/2\pi r$ for a constant current in a round solid copper rod. The solid curve is the total loss (electromagnetic plus shock) whereas the dashed line is the electromagnetic loss only.

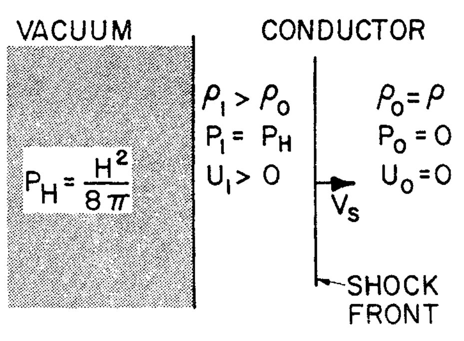

Figure 2. Shock compression losses in a current carrying conductor. The magnetic field is produced by the current flowing in the conductor. The quantities ρ, P, and u refer to the material density, pressure and velocity, respectively; the subscripts o,1 refer to the unshocked and shocked regions, respectively. V_S is the shock velocity.